Green’s function treatment of the localized antiferromagnetic system

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An extended Green’s function method is presented for a localized antiferromagnetic (AF) system with mixed AF and ferromagnetic couplings, and single-ion anisotropy. Expressions are given for the Néel temperature, the temperature-dependent magnetic order parameter (MOP), and the spin component along the preferred direction of the single-ion anisotropy in the ground state for A-type AF LaMnO$_3$. These predict the MOP found by inelastic neutron scattering. Our theory exhibits a nontrivial internal consistency with four measured parameters, including $T_N$. Our theory also predicts a temperature-dependent spin energy gap that is different from the Holstein-Primakoff theory.

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I. INTRODUCTION

The description of the magnetic properties of manganites is a fundamental need for understanding colossal magnetoresistance (CMR).\textsuperscript{1} Here, we present a theoretical treatment of a localized spin system that addresses two vital issues. First, in CMR materials, most of the $d$ electrons are localized on Mn ions. Previously, we developed a quantum form Hamiltonian for the double exchange (DE) interaction and showed that quantum localized spins appear in a linear form of the scalar product in the Hamiltonian that can be treated in a way similar to a pure Heisenberg magnet.\textsuperscript{2} Second, as De Gennes\textsuperscript{3} pointed out, in hole-doped manganites the DE interaction will compete with antiferromagnetic superexchange (SE). The latter can be written in a typical Heisenberg form.\textsuperscript{4} The usual solution of a Heisenberg magnet Hamiltonian is the Holstein-Primakoff (HP) “spin-wave amplitudes” theory,\textsuperscript{5} in which the localized spins are approximated by a set of coordinates that describe the quantum state of the system. In this method, the Hamiltonian of the system splits into quadratic terms in the amplitudes plus higher-order terms. The quadratic part gives a noninteracting spin-wave theory and the other part represents the effects of interactions between spin waves within perturbation theory. This method was also used by others.\textsuperscript{6} Dyson\textsuperscript{7} invented a rigorous theory to deal with spin-wave interactions.

However, the above theories treat the statistics of a spin system by a series expansion in powers of $T$ that is only valid at low temperatures. Callen\textsuperscript{8} invented a Green’s function theory for Heisenberg ferromagnets (FMs) that covers the whole temperature range. In this Green’s function formalism, the exact equation of motion involves higher-order Green’s functions than the initial one, and this is shown to lead to a chain of coupled equations for the higher-order Green’s functions.\textsuperscript{9} Callen’s method approximates the contributions of all higher-order Green’s functions by a coefficient that multiplies the initial Green’s function; this is equivalent to assuming all higher-order Green’s functions are proportional to the initial Green’s function. For convenience of our discussion, the proportional coefficient is hereafter called Callen’s decoupling parameter (CDP) (the CDP equals $\langle S_z \rangle / 2S^2$ for FM) and the method is called Callen’s decoupling approximation (CDA). Anderson and Callen\textsuperscript{10} extended the CDA for a FM to a pure antiferromagnet (PAF) in which all nearest-neighbor (NN) couplings are antiferromagnetic. This extension is suitable for describing a $G$-type AF.\textsuperscript{11} Moreover, these authors used the same CDP as in the FM case. Lee and Liu\textsuperscript{12} made a similar extension to a PAF with a negative CDP ($-\langle S_z \rangle / 2S^2$). Ghosh\textsuperscript{13} used a self-consistent moment-conserving decoupling method to deal with the PAF system that obtained a much higher Néel temperature than the above two extensions.

However, the CMR materials display richer magnetic phenomena in that their magnetic structures are not always PAF, even for completely localized spins.\textsuperscript{11} Moreover, in AF materials, the exact ground state of the Heisenberg Hamiltonian is not known; therefore, the sign and value of its CDP is not known either. It can be shown that both CDPs, $\langle S_z \rangle / 2S^2$ and $-\langle S_z \rangle / 2S^2$, lead to higher Néel temperatures than recent experimental ones,\textsuperscript{14} even if deviations from PAF are considered (this will be discussed below).

The present paper extends the CDA to the case of antiferromagnetism with mixed couplings (AF and FM), a different CDP, and includes single-ion anisotropy. A double-time Green’s function\textsuperscript{8,9} solution for the AF system is obtained. The results show that the sublattice spin component is proportional to its corresponding magnetization or magnetic order parameter (MOP). In addition, the Néel temperature of an AF system can be expressed in an explicit form that contrasts with the HP theory, which uses series expansions. Application of present model to LaMnO$_3$ shows that the theory describes the neutron scattering MOP data\textsuperscript{14} over the whole temperature range up to the Néel temperature. Our theory exhibits a nontrivial internal consistency with four measured parameters, including $T_N$. A temperature-dependent spin energy gap is predicted that is different from the HP theory.

II. GREEN’S FUNCTION METHOD FOR LOCALIZED ANTIFERROMAGNETISM

To describe the magnetic properties of AF materials with the single-ion magnetic anisotropy, we start with the following Hamiltonian:

$$H = \sum_i (\frac{1}{2} \epsilon_i S_i^2 + \frac{1}{4} g_{\text{an}} S_i^4) + \sum_{i<j} J_{ij} S_i \cdot S_j,$$

where $\epsilon_i$ is the single-ion anisotropy, $g_{\text{an}}$ is the single-ion anisotropy parameter, $J_{ij}$ is the exchange integral, and $S_i$ is the spin of the $i$th site.
where \( J(f, h) \) is the exchange constant between ions at sites \( f \) and \( h \), and the second sum represents single-ion anisotropy with a proportional coefficient \( D > 0 \). It is assumed that the \( J(f, h) \) may be either positive (AF coupling, with a subscript “a”) or negative (FM coupling, with a subscript “p”), but the ground state of the system is an antiferromagnetically aligned state and can be described by a two-sublattice model. Equation (1) implies that the magnetizations for the two sublattices are in positive and negative \( z \) directions that are hereafter labeled “u” and “d” sublattices, respectively.

The above Hamiltonian [Eq. (1)] allows us to apply the double-time Green’s function method \(^8\) to obtain the solution. The temperature-dependent retarded Green’s function

\[
(A(t); B) = -i \theta(t)[(A(t); B)]_t,
\]

where \( A \) and \( B \) are either Fermi or Bose operators and, correspondingly, the commutator is \([A, B]_t = AB - BA:\]

\[
\theta(t) = \begin{cases} 
1, & t > 0 \\
0, & t < 0.
\end{cases}
\]

Single angular brackets denote an average with respect to the canonical density matrix of the system at temperature \( T \). The equation of motion of \( (A(t); B) \) in Fourier form is

\[
E(\langle A; B \rangle)_E = \frac{1}{2\pi} [A, B]_t + \langle [\langle A(t); H \rangle; B]_t \rangle_E.
\]

The correlation function \( BA(t) \) can be extracted from the solution of the above equation through the relations

\[
\langle S_f^+ \rangle = \lim_{T \to 0} \int_{-\infty}^{\infty} \frac{\langle \langle A; B \rangle \rangle_{\text{harmonic}} - \langle \langle A; B \rangle \rangle_{\text{harmonic}} e^{-i\omega t}}{\exp(\hbar\omega/k_B T) - 1} d\omega,
\]

\[
\langle AB(t) \rangle = \lim_{T \to 0} \int_{-\infty}^{\infty} \frac{\langle \langle A; B \rangle \rangle_{\text{harmonic}} - \langle \langle A; B \rangle \rangle_{\text{harmonic}} e^{-i\omega t}}{\exp(\hbar\omega/k_B T) - 1} \times \exp(\hbar\omega/k_B T)e^{i\omega t} d\omega.
\]

In our calculation, we shall need two kinds of correlation functions, \( \langle S_{f}^{z}(t)S_{h}^{z}(t) \rangle \) and \( \langle S_{f}^{+}(t)S_{f}^{-}(t) \rangle \), and we shall therefore study spin Green’s function \( \langle S_{f}^{+}(t); e^{\pm\hbar\omega S_{h}^{z}}(t') \rangle \), where \( \omega \) is a parameter used for the thermodynamic average of the \( z \) component of the sublattice spin \( \langle S_{h}^{z} \rangle \) following Callen’s method.\(^8\) The equation of motion for the spin Green’s function is

\[
E(\langle S_{f}^{+}; e^{\pm\hbar\omega S_{h}^{z}} \rangle)_E = \frac{1}{2\pi} [\langle S_{f}^{+}, e^{\pm\hbar\omega S_{f}^{z}} \rangle]_t \delta_{h} + \langle [\langle S_{f}^{+}(t); H \rangle; e^{\pm\hbar\omega S_{h}^{z}}(t) \rangle \rangle_E.
\]

To conveniently describe the solutions of Eq. (1), we define \( \langle S_{u}^{z} \rangle \) and \( \langle S_{d}^{z} \rangle \) to represent the thermodynamic average of the \( z \) component of the sublattices “u” and “d,” respectively. Correspondingly, the Green’s functions for two cases are defined by

\[
\langle S_{f}^{+}; e^{\pm\hbar\omega S_{h}^{z}} \rangle_\tau = \begin{cases}
\tau = p, & f \text{ and } h \text{ are on the same sublattice} \\
\tau = a, & f \text{ and } h \text{ are on opposite sublattices}.
\end{cases}
\]

If the two sublattices are translationally invariant, we may perform the spatial Fourier transform with respect to the reciprocal sublattices for the Green’s functions

\[
\langle S_{f}^{+}; e^{\pm\hbar\omega S_{h}^{z}} \rangle_\tau = \begin{cases}
\frac{2}{N} \sum_{k} G_{pk} e^{i(\tau - h)k}, & f \text{ and } h \text{ are on the same sublattice} \\
\frac{2}{N} \sum_{k} G_{ak} e^{i(\tau - h)k}, & f \text{ and } h \text{ are on opposite sublattices},
\end{cases}
\]

where \( N \) is the number of magnetic atoms in the material and \( f \) and \( h \) are the atom positions \( R_{f} \) and \( R_{h} \), with the number of magnetic atoms in “u” and “d” sublattice being \( N/2 \). To transform Eq. (4) into a spatial Fourier form, we choose “h” to be on the “u” sublattice. We use Callen’s decoupling procedure

\[
\langle S_{f}^{+}; e^{\pm\hbar\omega S_{h}^{z}} \rangle = \sum_{\alpha} \langle S_{\alpha}^{+}; e^{\pm\hbar\omega S_{\alpha}^{z}} \rangle, \quad \langle S_{\alpha}^{+}; e^{\pm\hbar\omega S_{\alpha}^{z}} \rangle \rangle_{f^{+}} \tag{f+1}
\]

\[
- \alpha_{1} \langle S_{\alpha}^{+}; e^{\pm\hbar\omega S_{\alpha}^{z}} \rangle \rangle_{f^{+}} \langle S_{f}^{+}; e^{\pm\hbar\omega S_{h}^{z}} \rangle, \quad \langle S_{\alpha}^{+}; e^{\pm\hbar\omega S_{\alpha}^{z}} \rangle \rangle_{f^{+}} = \langle S_{\alpha}^{+}; e^{\pm\hbar\omega S_{\alpha}^{z}} \rangle \rangle_{f^{+}}, \quad \alpha_{1} = \langle S_{\alpha}^{+}; 2nS^{2} \rangle \text { is a correction to the Tyablikov decoupling} \tag{9} \]

to the Tyablikov decoupling with either a positive or negative sign and \( n = 1 \) in Callen’s FM theory. However, in an antiferromagnetic material, the exact ground state of the Heisenberg Hamiltonian is not known. Therefore, the sign and value of its Callen’s decoupling parameter is not known either. We argue that if a material can be described by the theory, its parameters derived by the theory must explain independent experiments on the material. Therefore, in the present case, we leave \( n \) to be a parameter that will be determined by fitting experimental data. Including translational invariance, the spatial Fourier transforms and the rearrangements of terms yield

\[
(E - B_{k}) G_{pk} = \frac{1}{2\pi} \Theta(u) A_{k} G_{ak}, \tag{5}
\]

\[
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\]

\[
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\]
where \( \Theta(w) = \langle 5^*; e^{wS^z}; 5^- \rangle \).

\[
B_k = 2[\langle S_\uparrow \rangle J_{\rho}(0) - \langle S_\downarrow \rangle J_{\rho}(0)] \\
+ 2 \frac{\langle S_\uparrow \rangle}{2nS^2N} \sum_{k'} [J_\rho(k-k')\Psi_\rho(k')] + 2\langle S_\rho \rangle J_{\rho}(k) \\
- 2 \frac{\langle S_\downarrow \rangle}{2nS^2N} \sum_{k'} [J_\rho(k-k')\Psi_\rho(k') + J_\rho(k')\Psi_\rho(k')] + 2D\langle S_\rho \rangle,
\]

(7)

\[
A_k = 2\langle S_\rho \rangle J_{\rho}(k) - 2 \frac{\langle S_\uparrow \rangle}{2nS^2N} \sum_{k'} [J_\rho(k-k')\Psi_\rho(k')],
\]

(8)

\[
\Psi_\rho(k) = \Theta(0) \left[ \frac{B_k}{2\sqrt{B_k^2 - A_k^2}} \coth \left( \frac{\sqrt{B_k^2 - A_k^2}}{2k_bT} \right) - \frac{1}{2} \right],
\]

(9)

\[
\Psi_\rho(k) = - \Theta(0) \left[ \frac{A_k}{2\sqrt{B_k^2 - A_k^2}} \coth \left( \frac{\sqrt{B_k^2 - A_k^2}}{2k_bT} \right) - \frac{1}{2} \right].
\]

(10)

The derivation of Eqs. (9) and (10) uses Eqs. (2)–(6) and \( \langle S_\uparrow \rho \rangle = - \langle S_\downarrow \rho \rangle = \langle S_\rho \rangle \). Following a standard procedure used by Callen, we obtain

\[
\langle S_\rho \rangle = \frac{(S - \Phi)(1 + \Phi)^{2S+1} + (S + 1 + \Phi)\Phi^{2S+1}}{(1 + \Phi)^{2S+1} - \Phi^{2S+1}},
\]

(11)

where

\[
\Phi = \frac{2}{N} \sum_k \frac{B_k}{2\sqrt{B_k^2 - A_k^2}} \coth \left( \frac{\sqrt{B_k^2 - A_k^2}}{2k_bT} \right) - \frac{1}{2}.
\]

(12)

From the definition of \( \Theta(w) \), it is easily shown that \( \Theta(0) = 2\langle S_\rho \rangle \). Note that our derivation of the coupled equations is not restricted to special crystal structure or limited to NN exchange interactions.

III. APPLICATION TO THE DESCRIPTION ON THE MAGNETIC PROPERTIES OF A MANGANITE

Recent inelastic neutron scattering experiments\(^{14}\) on LaMnO\(_3\) offer an opportunity to check the validity of the above theoretical model. To apply the theoretical model to the material, we shall take into account its magnetic structure and couplings. The experiments\(^{14}\) on LaMnO\(_3\) show that the material is an orthorhombic crystal with A-type AF order and magnetic moments along \( b \). The linear spin-wave spectrum analysis reveals that two kinds of exchange interactions: FM coupling in the basal plane \((a,b)\) and AF coupling along \( c \) together with NN interactions are needed to explain the dispersion curves in the experiments. Therefore, in our model we shall use the same magnetic structure and coupling as above, with the \( z \) direction along \( b \). Taking into account these factors and the crystal symmetry, Eqs. (7) and (8) can be transformed into

\[
B_k = 2z_2J_2\langle S_\rho \rangle R_a + 2z_1J_1\langle S_\rho \rangle R_p[1 - \gamma_\rho(k)] + 2D\langle S_\rho \rangle,
\]

(13)

\[
A_k = 2z_2J_2\langle S_\rho \rangle R_a \gamma_\rho(k),
\]

(14)

where

\[
z_1 = 4, \quad z_2 = 2, \quad \gamma_\rho(k) = \frac{1}{2}(\cos k_a a + \cos k_b b),
\]

\[
\gamma_\rho(k) = \cos k_c c,
\]

\[
R_a = 1 + \frac{\langle S_\rho \rangle}{2nS^2N} \sum_{k'} \gamma_\rho(k') \frac{A_k}{2E_k'} \coth \left( \frac{E_k'}{2k_bT} \right),
\]

\[
R_p = 1 + \frac{\langle S_\rho \rangle}{2nS^2N} \sum_{k'} \gamma_\rho(k') \left[ \frac{B_k'}{2E_k'} \coth \left( \frac{E_k'}{2k_bT} \right) - \frac{1}{2} \right],
\]

(15)

where \( J_1 \) and \( J_2 \) are NN exchange coupling constants in the basal plane \((a,b)\) and along \( c \), respectively. Thus, Eqs. (13), (14) plus (9)–(12) describe the material LaMnO\(_3\).

Having found the solution of the Hamiltonian (1) for LaMnO\(_3\), we now discuss some features of the present theoretical description. First, we derive the parameters \( J_1, J_2, \) and \( D \) in the spin-wave energies spectrum. If we compare Eqs. (13)–(15) with the corresponding results derived from HP’s linear spin-wave theory,\(^{14}\) we find that our spin-wave spectrum is renormalized by temperature-dependent coefficients. The parameters \( J_1 = 0.83 \) meV, \( J_2 = 0.58 \) meV, and \( D = 0.165 \) meV derived at 20 K in the experiments\(^{14}\) are equivalent to \( \langle S_\rho \rangle / S R_p J_1, \langle S_\rho \rangle / S R_p J_2, \) and \( \langle S_\rho \rangle / S D \) at the same temperature in our model. These will be used to deduce parameters \( J_1, J_2, \) and \( D \) in our model.

Next, we show the consistency for our model to describe the experiments. We follow Callen’s method and do the expansion analysis for Eqs. (11), (12), \( R_a, \) and \( R_p \) when the temperature approaches the Néel temperature \( T_N \) from below, and find the expression for \( T_N \). This procedure leads to

\[
\frac{k_bT_N}{J_2} = \frac{2z_2(S + 1)}{9nF_\rho(1)} \left[ 3nSF_\rho(1) + (S + 1)F_\rho(2) \right],
\]

(16)

where

\[
F_\rho(1) = \frac{1}{1 + \frac{z_1J_1R_pN}{z_2J_2R_aN}[1 - \gamma_\rho(k)]} + \frac{D}{z_2J_2R_aN},
\]

\[
F_\rho(2) = \frac{1}{1 + \frac{z_1J_1R_pN}{z_2J_2R_aN}[1 - \gamma_\rho(k)] + \frac{D}{z_2J_2R_aN}}^2 - \gamma_\rho^2(k),
\]

(17)
Using Eq. (16), the experimental parameters $J_1$, $J_2$, and $D$' for a Gaussian quadrature approximation for the sums over $\mathbf{k}$, and $T_N=139.5$ K from the experiment, we obtain parameters $J_1=0.85$ meV, $J_2=0.63$ meV, $D=0.167$ meV, and $n=-0.707$. Thus, the parameters $J_1$, $J_2$, and $D$ from our model are slightly modified from the experimental ones, but $n$ differs significantly from the FM case of Callen ($n=1$). What is important to emphasize here is the nontrivial internal consistency of the theory with the four experimental parameters, $J_1$, $J_2$, $D$, and $T_N$.

It is noteworthy that if we use the same CDP as in Ref. 10 or Ref. 12, $T_N$ would be 179.7 K (or 146.4 K). Next, we show the further consistency of this theory by using the above parameters to calculate the temperature-dependent MOP. In our calculation, the sublattice magnetization is $M_s=g\mu_B(S)$, where $g$ is Lande $g$-factor and $\mu_B$ is the Bohr magneton. The temperature dependence of $M_s$ (thus MOP) can be calculated by using Eqs. (11) and (12). Figure 1 shows the comparison of the square MOP $M_s^2$ from the present theory with experiment,\textsuperscript{14} It can be seen that the theoretical curve agrees with the experimental one when the temperature approaches $T_N$ and our theoretical critical exponent $\beta=0.47$. There is a discrepancy between theory and the experimental value of $\beta=0.28$. Adjusting $D$ could improve this, however, which would change the spin energy gap (see below) and contradict the inelastic neutron scattering data. Note that the parameter $D$ has a clear sample dependence,\textsuperscript{15} so that high quality single crystals are needed for a quantitative comparison. Unfortunately, the experimental sample is a twinned single crystal and the fit of dispersion curves is less than perfect.

Furthermore, due to the magnetic anisotropy [Eq. (1)], a spin energy gap $E_0=E_{\mathbf{k}k_{\mathbf{k}}}=2(S)^1/2(D+z_2J_2R_s)$ is predicted from Eqs. (13)–(15). This is a temperature-dependent gap that is different from the simple HP theory with a temperature-independent gap.

Finally, we show that the sublattice $z$ component $\langle S_{z0} \rangle$ of the spin per Mn ion in LaMnO$_3$ at ground state ($T=0$ K) can be directly derived. This can be done by using the limit $\lim_{\gamma \rightarrow 0} \coth(\gamma/x)=1$ ($y>0$), and thus we obtain

$$F_s(3)$$

$$\gamma_0(k) \left\{ \frac{z_1J_1R_{pN}}{z_2J_2R_{aN}}[1-\gamma_0(k)] + \frac{D}{z_2J_2R_{aN}} \right\}^2 - \frac{\gamma_0^2(k)}{4},$$

$$R_{aN} = R_a|_{T=T_N} = \frac{3nSF_s(1)+(S+1)F_s(2)}{3nSF_s(1)},$$

$$R_{pN} = R_p|_{T=T_N} = \frac{3nSF_s(1)+(S+1)F_s(3)}{3nSF_s(1)}.$$

Using Eq. (16), the experimental parameters $J'_1$, $J'_2$, and $D'$, a Gaussian quadrature approximation for the sums over $\mathbf{k}$, and $T_N=139.5$ K from the experiment, we obtain parameters $J_1=0.85$ meV, $J_2=0.63$ meV, $D=0.167$ meV, and $n=-0.707$. Thus, the parameters $J_1$, $J_2$, and $D$ from our model are slightly modified from the experimental ones, but $n$ differs significantly from the FM case of Callen ($n=1$). What is important to emphasize here is the nontrivial internal consistency of the theory with the four experimental parameters, $J_1$, $J_2$, $D$, and $T_N$. It is noteworthy that if we use the same CDP as in Ref. 10 or Ref. 12, $T_N$ would be 179.7 K (or 146.4 K). Next, we show the further consistency of this theory by using the above parameters to calculate the temperature-dependent MOP. In our calculation, the sublattice magnetization is $M_s=g\mu_B(S)$, where $g$ is Lande $g$-factor and $\mu_B$ is the Bohr magneton. The temperature dependence of $M_s$ (thus MOP) can be calculated by using Eqs. (11) and (12). Figure 1 shows the comparison of the square MOP $M_s^2$ from the present theory with experiment.\textsuperscript{14} It can be seen that the theoretical curve agrees with the experimental one when the temperature approaches $T_N$ and our theoretical critical exponent $\beta=0.47$. There is a discrepancy between theory and the experimental value of $\beta=0.28$. Adjusting $D$ could improve this, however, which would change the spin energy gap (see below) and contradict the inelastic neutron scattering data. Note that the parameter $D$ has a clear sample dependence,\textsuperscript{15} so that high quality single crystals are needed for a quantitative comparison. Unfortunately, the experimental sample is a twinned single crystal and the fit of dispersion curves is less than perfect.

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$$R_{aN} = R_a|_{T=T_N} = \frac{3nSF_s(1)+(S+1)F_s(2)}{3nSF_s(1)},$$

$$R_{pN} = R_p|_{T=T_N} = \frac{3nSF_s(1)+(S+1)F_s(3)}{3nSF_s(1)}.$$
ever, the present case offers a direct way to estimate $\langle S_z \rangle$ for a system with AF and FM mixed couplings, and it agrees with the value of $\langle S_z \rangle = 1.935$ that was deduced from Rietveld refinement of neutron scattering data at $T = 1.4$ K$^{18}$ by setting $g \sim 2.0$.

IV. CONCLUDING REMARK

We have extended Callen’s Green’s function method to a localized AF with mixed couplings (AF and FM), and included the existence of single-ion anisotropy. This theory is consistent with four experimental parameters, including the Néel temperature, the temperature dependence of MOP, and the spin component along the preferred direction of the single-ion anisotropy at the ground state for LaMnO$_3$. Thus, it describes the inelastic neutron scattering data of the material very well. Our theory exhibits a nontrivial internal consistency with four measured parameters, including $T_N$. It also predicts a temperature-dependent spin energy gap that is different from the simple HP theory.

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7 F. J. Dyson, Phys. Rev. 102, 1217 (1956); 102, 1230 (1956).
16 The sum may be replaced by an integral over the first Brillouin zone. The integral can be approximately transformed into Gaussian quadrature summation; we can then find numerical solutions for the parameters. The Gaussian quadrature method can be found in S. C. Chapra and R. P. Canale, Numerical Methods for Engineers, Chap. 22 (China Science Press and McGraw-Hill, Beijing, 2000).
18 This temperature is much less than the Néel temperature; therefore, the corresponding experimental $\langle S_z \rangle$ is expected to be approximately the experimental value in the ground state.